



I can, without a calculator, use trigonometric identities such as angle addition/subtraction and double angle formulas, to express values of trigonometric functions in terms of rational numbers and radicals.

Simplify. Exact values only.

$$1. \frac{\tan(20^\circ) + \tan(10^\circ)}{1 - \tan(20^\circ)\tan(10^\circ)}$$

$$= \tan(20+10) = \tan(30) = \boxed{\frac{1}{\sqrt{3}}}$$

$$3. \sin(80^\circ)\cos(20^\circ) - \cos(80^\circ)\sin(20^\circ) =$$

$$= \sin(80-20) = \sin 60^\circ = \boxed{\frac{\sqrt{3}}{2}}$$

$$5. \sin(105^\circ)$$

$$= \sin(60+45)$$

$$= \sin(60) \cdot \cos(45) + \cos(60) \cdot \sin(45)$$

$$= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} + \frac{1}{2} \cdot \frac{\sqrt{2}}{2}$$

$$= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \boxed{\frac{\sqrt{6}+\sqrt{2}}{4}}$$

$$2. \sin(160^\circ)\cos(50^\circ) + \cos(160^\circ)\sin(50^\circ) =$$

$$= \sin(160+50) = \sin(210) = \boxed{-\frac{1}{2}}$$



$$4. \tan(195^\circ)$$

$$= \tan(150^\circ + 45^\circ)$$

$$= \frac{\tan(150) + \tan(45)}{1 - \tan(150)\tan(45)} = \frac{-\frac{1}{\sqrt{3}} + 1}{1 - \frac{1}{\sqrt{3}} \cdot 1}$$

$$6. \sin\left(\frac{17\pi}{12}\right) =$$

$$= \sin\left(\frac{9\pi}{12} + \frac{8\pi}{12}\right)$$

$$= \sin\left(\frac{3\pi}{4} + \frac{2\pi}{3}\right)$$

$$= \sin\frac{3\pi}{4} \cdot \cos\frac{2\pi}{3} + \cos\frac{3\pi}{4} \cdot \sin\frac{2\pi}{3}$$

$$= \frac{\sqrt{2}}{2} \cdot -\frac{1}{2} + -\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2}$$

$$= -\frac{\sqrt{2}}{4} - \frac{\sqrt{2} \cdot \sqrt{3}}{4}$$

$$= \boxed{-\frac{\sqrt{2} + \sqrt{6}}{4}}$$

$$7. \sin(\alpha + \beta) - \sin(\alpha - \beta) =$$

$$= \cancel{\sin \alpha \cos \beta + \cos \alpha \sin \beta} - (\cancel{\sin \alpha \cos \beta} - \cancel{\cos \alpha \sin \beta})$$

$$= \boxed{2 \cos \alpha \sin \beta}$$

8. Given:  $\sin \alpha = \frac{3}{5}$ ,  $\cos \beta = \frac{4}{9}$ ,  $\alpha$  in quadrant II,  $\beta$  in quadrant IV

Find the following:

- $\sin(\alpha + \beta)$
- $\cos(\alpha + \beta)$
- $\tan(\alpha + \beta)$

$$\sin \alpha = \frac{3}{5}, \cos \alpha = -\frac{4}{5}$$

$$\sin \beta = -\frac{\sqrt{65}}{9}, \cos \beta = \frac{4}{9}$$

a)  $\sin(\alpha + \beta)$

$$= \frac{3}{5} \cdot \frac{4}{9} + -\frac{4}{5} \cdot -\frac{\sqrt{65}}{9}$$

$$= \frac{12}{45} + \frac{4\sqrt{65}}{45}$$

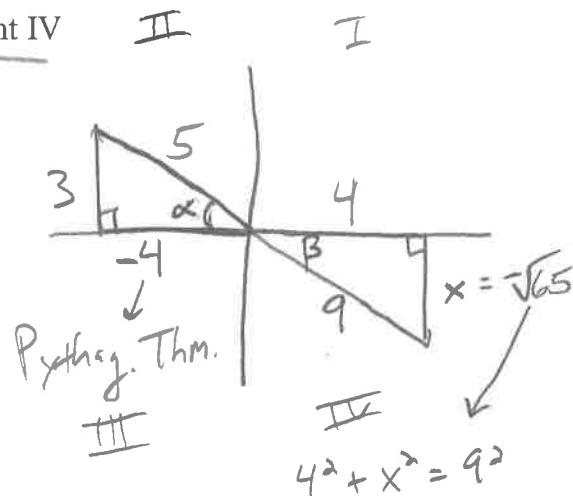
$$= \boxed{\frac{12 + 4\sqrt{65}}{45}}$$

b)  $\cos(\alpha + \beta)$

$$= -\frac{4}{5} \cdot \frac{4}{9} - \frac{3}{5} \cdot -\frac{\sqrt{65}}{9}$$

$$= \frac{-16}{45} + \frac{3\sqrt{65}}{45}$$

$$= \boxed{\frac{-16 + 3\sqrt{65}}{45}}$$



$$x^2 + x^2 = 9^2$$

$$x^2 = 65$$

$$x = \sqrt{65}$$

c)  $\tan(\alpha + \beta) = \frac{\sin(\alpha + \beta)}{\cos(\alpha + \beta)}$

$$= \frac{12 + 4\sqrt{65}}{45}$$

$$= \frac{-16 + 3\sqrt{65}}{45}$$

$$= \boxed{\frac{12 + 4\sqrt{65}}{-16 + 3\sqrt{65}}}$$

9.  $\sin\left(\frac{\pi}{2} + x\right) = \cos x$

$$= \sin \frac{\pi}{2} \cdot \cos x + \cos \frac{\pi}{2} \cdot \sin x$$

$$= (1) \cdot \cos x + (0) \cdot \sin x$$

$$= \boxed{\cos x}$$

10.  $\sin(\pi - x) = \sin x$

$$= \sin(\pi) \cdot \cos x - \cos \pi \cdot \sin x$$

$$= (0) \cdot \cos x - (-1) \cdot \sin x$$

$$= \boxed{\sin x}$$